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The report is divided into two parts. The first concerns the work on (i) numerical methods for first order nonlinear equations PDE, deterministic control and large deviations, (ii) development of the Skorokhod Problem and related methods for analyzing and controlling queueing and service networks, and (iii) variational and stochastic methods in image recognition, and the first PI was the main contributor. The second part, in which the second PI was the main contributor concerned (i) nonlinear filtering, (ii) communications systems, and (iii) stochastic algorithms for adaptive communications systems.

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Final Report ARO Contract DAAH04-96-1-0075
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The report is divided into two parts. The first concerns the work on (i) numerical methods for first order nonlinear equations PDE, deterministic control and large deviations, (ii) development of the Skorokhod Problem and related methods for analyzing and controlling queueing and service networks, and (iii) variational and stochastic methods in image recognition, and the first PI was the main contributor. The second part, in which the second PI was the main contributor concerned (i) nonlinear filtering, (ii) communications systems, and (iii) stochastic algorithms for adaptive communicatios systems.

Part 1. There are a number of deterministic optimal control problems for which a global approximation to the value function is needed. For example, in small noise risk-sensitive and robust nonlinear filtering, the optimal (robust) filter is defined in terms of the value function for a calculus of variations problem in which the variational integrand is a functional of the observation process. Other examples occur in computer vision and related areas, where the optimization problem appears as a representation for an unknown shape or other geometric object, and in large deviations, where the solution is used to construct an approximation to an invariant distribution. In these examples the representation usually takes the form of a calculus of variations problem, although in some cases a differential game representation is required. In other problems a global approximation is needed so that an approximation to the optimal control can be defined at all points in the state space. Examples from this category are recent formulations of robust nonlinear control, in which the value function is defined in terms of a differential game rather than an optimal control problem (although in some cases it can also have a representation in terms of an optimal control problem). An common special case for all these problems is that in which the dynamics are linear in the control and the cost is quadratic in the control (although general nolinearities as a function of the state are allowed). The paper [1] constructs Markov chain approximations for this important class of deterministic control problems and differential games. The emphasis is on the construction of schemes that are easily programmed and which possess a number of highly desirable qualitative properties. In particular, (i) the schemes have minimal numerical dissipativity; (ii) all minimizations required by the iterative solvers which calculate the approximation can be done analytically; and (iii) the iterative solvers converge rapidly. Computational examples are given, as is a representative proof of convergence.

The development of numerical methods for this class of problems is very active. The main competitor with our algorithms is the so-called *fast marching method*. This method has been developed extensively by Sethian, but in fact appears in an earlier paper of Tsitsiklis. Our method is easier to code and applies to a much broader class of problems. In terms of computational performance, the computation associated with the fast marching method scales as $N \log N$, where $N^3 D n^d$ is the total number of grid points, d is the dimension, and $1/n$ is proportional to the discretization in each coordinate direction. In contrast, the algorithms we have constructed typically scale as N.

Further development of our method to higher order accurate algorithms appears in [2] and [3]. The paper [2] proves foundational results on the convergence of controls and gradients of value functions, which are then used in [3] to construct and prove the convergence and higher

order accuracy of a second order scheme. The computational burden of this higher order scheme also scales as n.

Other work carried out during the period has concentrated on the theory and applications of the Skorokhod Problem. The solution to the Skorokhod Problem provides the correct input/output map for many models of communication and queueing systems, and could be considered the analogue of the linear system model for standard systems theory (i.e., systems without constraints). For many models are currently (formally) used for which little is known regarding existence, regularity, and approximation properties of the corresponding input/output map. The paper [4] develops a comprehensive approach to this problem that is based on ideas from convex duality. The limitations of standard methods are identified in very explicit terms, and new techniques for analyzing systems not covered by these results. Two of the main applications of this theory have been to the development of control techniques for queueing and service networks and to analyzing large deviations in stochastic networks [5].

In the past few years a new approach has emerged for analyzing the stability properties of constrained stochastic processes, such as reflecting Brownian motion, reflecting diffusions, and queueing networks. In this approach, one associates with the stochastic model a deterministic model (or a family of deterministic models), and, under appropriate conditions, stability of the stochastic model follows if all solutions of the deterministic model are attracted to the origin. In the paper [6] it is shown that a sharp characterization for the stability of the deterministic model is possible when it can be represented in terms of a "regular" Skorokhod Map. This characterization is as explicit as the classical eigenvalue characterization of stability for linear systems, and is currently being used in the explicit construction of stabilizing and optimally stabilizing service and routing controls for networks.

This paper [7] contains two main results. The first is a variational representation for the expectation of a measurable function of a Hilbert space valued Brownian motion, when the function is uniformly positive and bounded from above and the Brownian motion has a trace class covariance. This representation is then applied to derive the second main result, which is the large deviation principle for a class of Hilbert space valued diffusions with small noise. This result has applications to the study of image matching problems from a Bayesian perspective, and in particular to the derivation of approximate maximum a posteriori estimators in the small noise limit.

[1] Markov Chain Approximations for Deterministic Control Problems: Linear Dynamics in the Control and Quadratic Cost, to appear in *SIAM J. on Numerical Analysis.*

[2] Convergence of the optimal feedback policies in a numerical method for a class of deterministic optimal control problems, submitted to *SIAM J. on Control and Opt.*

[3] Second Order Numerical Methods for First Order Hamilton-Jacobi Equations, submitted to *SIAM J. on Numerical Analysis.*

[4] Convex Duality and the Skorokhod Problem, parts I and II, to appear in *Prob. Th. and Rel. Fields.*

[5] Large deviations and queueing networks: methods for rate function identification, to appear

in *Stoch. Proc. and Their Appl.*

[6] Simple necessary and sufficient conditions for the stability of constrained processes, to appear in *SIAM J. on Applied Math.*

[7] Representations and large deviations for Hilbert space valued diffusions.

Part 2. Work on nonlinear filtering. We are concerned with approximations to nonlinear filtering problems that are of interest over a very long time interval [1]. Since the optimal filter can rarely be constructed, one needs to compute with numerically feasible approximations. The signal model can be a jump–diffusion or just a process that is approximated by a jump–diffusion. The observation noise can be either white or of wide bandwidth. The observations can be taken either in discrete or continuous time. The cost of interest is the pathwise error per unit time over a long time interval. It was shown [1] under quite reasonable conditions on the approximating filter and the signal and noise processes that (as time, bandwidth, process and filter approximation, etc.) go to their limit in any way at all, the limit of the pathwise average costs per unit time is just what one would get if the approximating processes were replaced by their ideal values and the optimal filter were used. Analogous results are obtained (with appropriate scaling) if the observations are taken in discrete time, and the sampling interval also goes to zero. For these cases, the approximating filter is a numerical approximation to the optimal filter for the presumed limit (signal, observation noise) problem.

When suitable approximating filters cannot be readily constructed due to excessive computational requirements or to problems associated with a high signal dimension, approximations based on random sampling methods (or, perhaps, combinations of sampling and analytical methods) become attractive, and are the subject of a great deal of attention. This is somewhat analogous to the use of monte carlo methods for high dimensional integration problems. Owing to the sampling errors as well as to the other (computational and modeling) approximations that are made, in the filter and signal processes, it is conceivable that the long term pathwise average errors per unit time will be large, even with approximations that would perform well over some bounded time interval. The work of the previous work was extended [2] to a wide class of such algorithms. Under quite broad conditions, covering virtually all the cases considered to date, it was shown that the pathwise average errors converge to the same limit that would be obtained if the optimal filter were used, as time goes to infinity and the approximation parameter goes to its limit in any way at all. It can be extended to cover wide bandwidth observation or system driving noise as well.

An effective form of the Gaussian or moment approximation method for approximating optimal nonlinear filters with a diffusion signal process and discrete time observations was developed [3]. Various computational simplifications reduce the dimensionality of whatever numerical integrations need to be done. This, combined with an iterative Gaussian quadrature method make the filter effective for real time use. The advantages are illustrated by a model which captures the general flavor of modeling the highly uncertain behavior of an object such as a ship near obstacles such as a shore line into which it cannot go, and must maneuver away in some unknown fashion. The observations are of very poor quality, yet the filter behaves well and is quite stable. It performs than all current alternatives to which it was compared, sometimes substantially better

A large part of the work in communications an computer networks is concerned with making assignments in an optimal or very good way. The work [4] was concerned with the optimal control of the assignment of jobs from several arriving random streams to one of a bank of processors. Owing to the difficulty of the general problem, a heavy traffic approach is used. The required work depends on the processor to which it is assigned. The information that the assignment can be based on is quite flexible, and several information structures (data on which the control is based) are considered. The assignment can be made on arrival or when the job is to be processed. There can be bursty arrivals (the bursts depending on randomly varying environmental factors), rather general nonlinear cost functions and other complications. It was shown, under reasonably general conditions, that the optimal costs for the physical systems converge to the optimal cost for the heavy traffic limit problem, as the heavy traffic parameter goes to its limit. Numerical data is presented to illustrate some of the potential uses of the limit process for obtaining optimal controls, or controls satisfying optimal tradeoffs among competing criteria. The methods of proof are quite powerful tools for such optimal control problems.

Communications systems often have many types of users. Since they share the same resource, there is a conflict in their needs. This conflict leads to the imposition of controls on admission or elsewhere. In this work [5], there are two types of customers, GP (Guaranteed Performance) and BE (Best Effort). We consider an admission control of GP customer which has two roles. First, to guarantee the performance of the existing GP customers, and second, to regulate the congestion for the BE users. The optimal control problem for the actual physical system is difficult. A heavy traffic approximation was used, with optimal or nearly optimal controls. It was shown that the optimal values for the physical system converge to that for the limit system and that good controls for the limit system are also good for the physical system. This was done for both the discounted and average cost per unit time cost criteria. Additionally, asymptotically, the pathwise average (not mean) costs for the physical system are nearly minimal when good nearly optimal controls for the limit system are used. Numerical data show that the heavy traffic optimal control approach can lead to substantial reductions in waiting time for BE with only quite moderate rejections of GP, under heavy traffic. It also shows that the controls are often linear in the state variables. The approach has many advantages. It is robust, simplifies the analysis (both analytical and numerical) and allows a more convenient study of the parametric dependencies. Even if optimal control is not wanted, the approach is very convenient for a systematic exploration of the possible tradeoffs among the various cost components. This is done by numerically solving a series of problems with different weights on the costs. We can then get the best tradeoffs, and the control policies which give them.

The work on adaptive stochastic algorithms in communications [6] was motivated by some general problems when game theoretic considerations enter in competitive problems. The following is a canonical problem. Customers arrive at a service center and must choose between two types of service; a channel that is shared by all currently in it, and a dedicated line. The mean service cost (or time) for any customer entering the shared resource depends on the decisions of all future arrivals up to the time of departure of that customer, and so has a competitive aspect. The system keeps a record of the (discounted) mean sample service costs (or times) for the shared resource, as a function of the number there when a new arrival joins, and the arriving customers use this to make their decisions. The decision rule of each arriving customer is based on its own immediate self interest, given the available data on the past performance. They select the service with the smallest estimated service cost. But, if the current estimate

of the cost for the shared resource equals that of the dedicated line, any decision is possible. The procedure is a type of learning algorithm. The long term behavior of the arrivals and its effect on the system averages is of interest. The convergence problem is one in asynchronous stochastic approximation, where the ODE might have a set-valued right hand side; i.e., it might be a differential inclusion. The set value arises due to the arbitrariness of the decision at certain values of the current estimates. It was shown that, asymptotically, the performance of the learning system is that for the symmetric Nash strategy, despite the allowed arbitrariness and lack of coordination.

[1] Approximation and Limit Results for Nonlinear Filters Over an Infinite Time Interval, *SIAM J. on Control* 1999.

[2] Approximation and Limit Results for Nonlinear Filters Over an Infinite Time Interval: Part II, Random Sampling Algorithms, to appear *SIAM J. on Control.*

[3] A Nonlinear Filtering Algorithm Based on an Approximation of the Conditional Distribution, to appear *IEEE Trans on Automatic Control.*

[4] Optimal Control of Assignment of Jobs to Processors Under Heavy Traffic, to appear *Stochastics.*

[5] Admission Control for Combined Guaranteed Performance and Best Effort Communications Systems Under Heavy Traffic, to appear *SIAM J. on Control.*

[6] Stochastic Approximation and User Adaptation in a Competitive Resource Sharing System, to appear *IEEE Trans on Automatic Control.*